BEGINNER'S BOX-1

1. $W = Fd\cos\theta \Rightarrow \cos\theta = \dfrac{W}{Fd}$

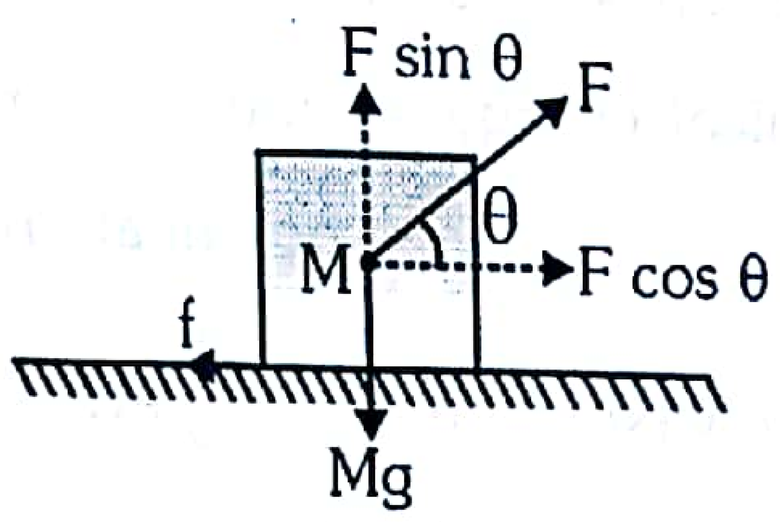
$W = 25$ J, $F = 5$ N & $d = 10$ m

so $\cos\theta = \dfrac{25}{5\times10} = \dfrac{1}{2} \Rightarrow \theta = 60°$

2. Work has to be done against gravity in second case.

3. From the figure $F\sin\theta + N = Mg$

$\therefore$ $N = Mg - F\sin\theta$



$F\cos\theta = f = \mu N = \mu[Mg - F\sin\theta]$

$F(\cos\theta + \mu\sin\theta) = \mu Mg$

$\therefore F = \dfrac{\mu Mg}{\cos\theta + \mu\sin\theta}$ = force required to pull an object

Work done in pulling an object

$W = Fd = \dfrac{\mu Mgd\cos\theta}{\cos\theta + \mu\sin\theta}$

4. $W = \int_{x_1}^{x_2} F\,dx = \int_{x_1}^{x_2} 2x\,dx$

$\Rightarrow W = x_2^2 - x_1^2$

5. $W = \int_0^2 F\,dx = \int_0^2 (10 + 0.5x)\,dx$

$\Rightarrow W = 10[x]_0^2 + 0.5\left[\dfrac{x^2}{2}\right]_0^2$

$\Rightarrow W = 21$ J

6. $v = ax^{3/2}$

$\Rightarrow \dfrac{dv}{dx} = \dfrac{3}{2}ax^{1/2}$

So $a = \dfrac{v\,dv}{dx} = \dfrac{3}{2}a^2x^2$

$F = ma = \dfrac{3}{4}a^2x^2$

$W = \int_0^2 F\,dx = \dfrac{3}{4}a^2\int_0^2 x^2\,dx$

$\Rightarrow W = 50$ J

7. $W = \vec{F}.\vec{x}$

$\vec{F} = 4\hat{i} + \hat{j} + 3\hat{k}$ N & $\vec{x} = \vec{r}_2 - \vec{r}_1$

$\Rightarrow \vec{x} = 11\hat{i} + 11\hat{j} + 15\hat{k}$

So $W = 100$ J

8. $W_1 = \int_0^a F_x\,dx = \int_0^a Kx\,dx = \dfrac{Ka^2}{2}$

$W_2 = \int_0^a F_y\,dy = \int_0^a Ky\,dy = \dfrac{Ka^2}{2}$

$W = W_1 + W_2 = \dfrac{Ka^2}{2} + \dfrac{Ka^2}{2} = Ka^2$

9. Resultant of three given forces –

$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$

$= 3\hat{i} + 9\hat{j} + 3\hat{k} + \left[-\dfrac{10}{7}\hat{i} + \dfrac{45}{7}\hat{k}\right] + [22\hat{i} + 11\hat{j} + 66\hat{k}]$

$= \left[\dfrac{165}{7}\hat{i} + 20\hat{j} + \dfrac{528}{7}\hat{k}\right]$ units

Displacement $\vec{r} = \vec{r}_2 - \vec{r}_1$

$= [11\hat{i} + 6\hat{j} + 8\hat{k}] - [4\hat{i} - 1\hat{j} + 1\hat{k}]$

$= (7\hat{i} + 7\hat{j} + 7\hat{k})$ units

work done $= \vec{F}.\vec{r}$

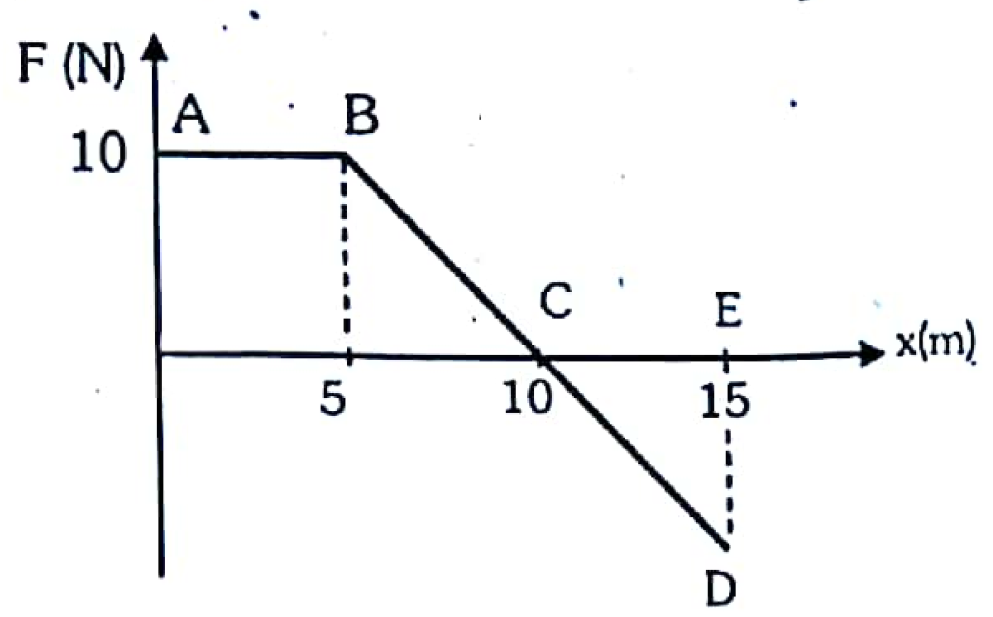
$= \left[\dfrac{165}{7}\hat{i} + 20\hat{j} + \dfrac{528}{7}\hat{k}\right]; [7\hat{i} + 7\hat{j} + 7\hat{k}]$

$= 165 + 140 + 528 = 833$ unit

10. In rectilinear motion work done by a force equals to area between the force-position graph and the position axis

(a) $W_{0\to10}$ = Area of trapazium OABC = 75 J

(b) $W_{10\to15}$ = –Area of triangle CDE = –25 J



(c) $W_{0\to15}$ = Area of trapazium OABC – Area of triangle CDE = 50 J

BEGINNER'S BOX-2

1. $K = \frac{p^2}{2m} \Rightarrow K \propto p^2$

$\Rightarrow \frac{K_f}{K_i} = \frac{p_f^2}{p_i^2} = (1.5)^2 = 2.25$

$\Rightarrow K_f = 2.25\ K_i$

$\Rightarrow \frac{\Delta K}{K} = \frac{K_f - K_i}{K_i} = 125\%$

So kinetic energy increases by 125%

2. Let p = x so K = 3x

Now $K = \frac{1}{2}mv^2 = \frac{1}{2}p.v$

$\Rightarrow v = \frac{2K}{p} = 6$ units

3. According to work energy theorem

$W_{gravity} + W_{air} = \Delta KE \Rightarrow mgh + W_{air} = \frac{1}{2}mv^2 - 0$

$\Rightarrow 10 \times [10] \times [20] + W_{air} = 500$

$\Rightarrow$ Work done by air on object $W_{air} = -1500$ J

4. $\because x = \frac{t^3}{3}$ $\therefore v = \frac{dx}{dt} = t^2$, Velocity at t=0, u=0 and at t=1s v=1 m/s

Using work energy theorem : $W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

$= \frac{1}{2}1(1)^2 = 0.5$ J

BEGINNER'S BOX-3

1. (A) – p, r, s, (B) – t

2. **For (A) :** $p = \sqrt{2mK}$ if K ↑ then p ↑

For (B) : Its height may ↑ or ↓

For (C) : W = ΔK if ΔK = positive then W = positive

For (D) : The resultant force on the particle must be at an angle less than 90° all times

3. Mechanical energy = kinetic energy + potential energy

$E = K + U(x)$ where $K = \frac{1}{2}mv^2$

If K = 0 then E = U(x)

If F = 0 then $F = -\frac{dU(x)}{dx} = 0 \Rightarrow \frac{dU(x)}{dx} = 0$

4. (A) – q, (B) – r, (C) – p

5. $W_C + W_{nc} + W_{ext} = \Delta K$

$mgh - f.s + 0 = 0 \Rightarrow mgh - \mu mg.s = 0$

$\Rightarrow s = \frac{h}{\mu} = \frac{1}{0.2} = 5m$

BEGINNER'S BOX-4

1. $E = \frac{1}{2}kx^2$

If E = constant then $x \propto \frac{1}{\sqrt{k}}$

So $\frac{F_1}{F_2} = \frac{k_1}{k_2}.\frac{x_1}{x_2} = \frac{k_1}{k_2}\sqrt{\frac{k_2}{k_1}}$

$\Rightarrow \frac{F_1}{F_2} = \sqrt{\frac{k_1}{k_2}}$

2. In 1st situation $W = \frac{1}{2}k(1)^2$

& In 2nd situation $W' = \frac{1}{2}k(2)^2 = 4W$

So required work done = 4W – W = 3W

3. Let maximum compression is x_m

Using law of conservation of mechanical energy

$$mg(h + x_m) = \frac{1}{2}kx_m^2$$

$$\Rightarrow \quad 20(4 + x_m) = \frac{1}{2}1960\, x_m^2$$

$$\Rightarrow \quad 980\, x_m^2 - 20\, x_m - 80 = 0$$

$$\Rightarrow \quad 49x_m^2 - x_m - 4 = 0$$

4. By applying work energy theorem

$$\Delta K.E = W_s + W_{ext}$$

$$0 = -\frac{1}{2}Kx^2 + Fx \Rightarrow x = \frac{2F}{K}$$

$$\text{Work done} = \frac{2F^2}{K}$$

5. By applying work energy theorem

$$\frac{1}{2}m\frac{v^2}{4} - \frac{1}{2}mv^2 = -\frac{1}{2}kx^2$$

$$\Rightarrow \frac{-3mv^2}{8} = \frac{-1}{2}kx^2;\ k = \frac{3mv^2}{4x^2}$$

6. At maximum speed net force = 0

Applied force by engine = resistive forces

Here power of engine = constant

So $(6m)20 = (14m)v_1 = (8m)v_2$

$\Rightarrow \quad v_1 = 8.5$ m/s

and $v_2 = 15$ m/s

7. COME $\Rightarrow K_1 + U_1 = K_2 + U_2$

$$\Rightarrow 0 + \frac{1}{2}k_1x^2 + \frac{1}{2}k_2x^2$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}k_1\left(\frac{x}{2}\right)^2 + \frac{1}{2}k_2\left(\frac{x}{2}\right)^2$$

$$\Rightarrow \frac{1}{2}(k_1 + k_2)x^2 = \frac{1}{2}mv^2 + \frac{1}{8}(k_1 + k_2)x^2$$

$$\Rightarrow v = \sqrt{\frac{3}{4}\frac{(k_1 + k_2)x^2}{m}}$$

8. Output power of motor

$$= \frac{mgh}{t} = \frac{(30 \times 1000) \times 9.8 \times 40}{15 \times 60}$$

$$\because \quad \%\ \text{efficiency} = \frac{\text{Output power of motor}}{\text{Power consumed by motor}}$$

$$\Rightarrow \quad 30 = \frac{30 \times 1000 \times 9.8 \times 40}{15 \times 60 \times P}$$

$$\Rightarrow \quad P = \frac{9.8 \times 1000 \times 40}{15 \times 60} = 43.55 \times 10^3 W$$

$= 43.6$ kW

9. Output power of pump $P = \frac{mgh}{t} = \frac{100 \times 10 \times 10}{5}$

$P_{output} = 2$ kW

therefore, $P_{input} = \frac{P_{output}}{\eta} = \frac{2}{0.6} = 3.33$ kW

EXERCISE-I (Conceptual Questions) — Build Up Your Understanding

1. $W = \vec{F}.\vec{d}$

$= (2\hat{i} - \hat{j} + 4\hat{k}).(3i + 2\hat{j} - \hat{k}) = 0$

2. $\vec{d} = \vec{r}_2 - \vec{r}_1$

$\Rightarrow \vec{d} = \hat{i} + 2j + \hat{k}$ & $\vec{F} = (3\hat{i} + 2\hat{j} + 4\hat{k})$

So $W = \vec{F}.\vec{d} = 11$ J

3. $WD = \int_0^5 (3x^2 + 2x - 7)dx$

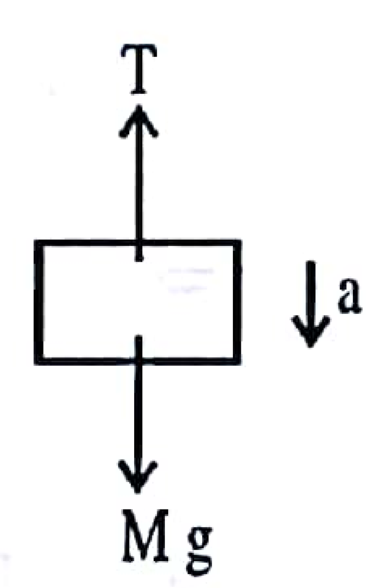
$= 125 + 25 - 35 = 115$ J

4. Work will be zero as angle between force & displacement is 90°

5. Force ⊥ displacement ⇒ WD = 0

as $\cos\theta = 0$

6. For the block

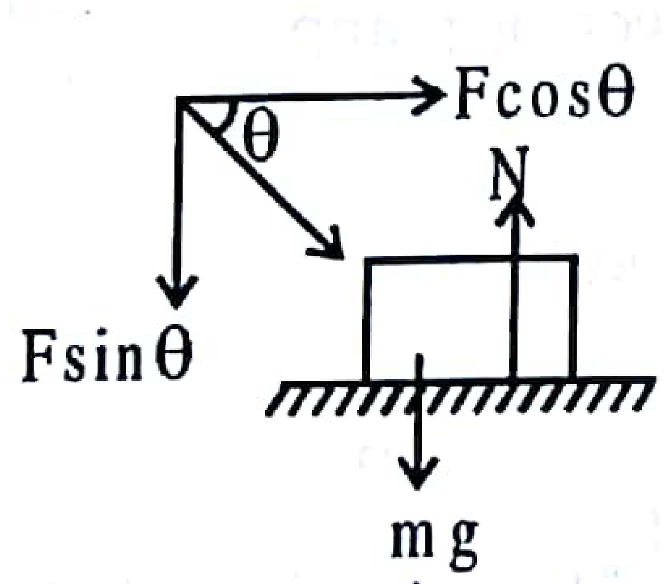


$Mg - T = M(g/4)$

$\Rightarrow T = \frac{3}{4}Mg$

So, Work $= \frac{3}{4}Mg(h)\cos 180° = -\frac{3}{4}Mgh$

7. $N = Mg + F\sin\theta$



$\vec{F}_{Friction} = \mu_k(mg + F\sin\theta)$

Work $= -\mu_k(mg + F\sin\theta)\, s$

8. Work done = Area under F – d curve

9. $dW = kx^2 dx \cos 60°$

$\therefore \quad WD = \frac{k}{2}\int_{x_1}^{x_2} x^2 dx = \frac{k}{6}\left(x_2^3 - x_1^3\right)$

10. $WD = \Delta PE = 10 \times 9.8 \times 1 = 98J$

11. $W = Fd\cos\theta$

$= 10 \times 10 \times \cos\theta = 50$

$\Rightarrow \quad \cos\theta = \frac{1}{2}$

$\Rightarrow \quad \theta = 60°$

12. $s = \frac{t^2}{4} \qquad v = \frac{t}{2}$

$t = 0, u = 0$

$t = 2, v = 1$

$\therefore \quad WD = \Delta KE = \frac{1}{2} \times 6 \times 1 = 3J$

13. $WD = 30 - 20 = 10J$

14. Area of graph $= [3 \times 3] + \left[\frac{1}{2} \times 3 \times 3\right] = 9 + 4.5$

$= 13.5J$

15. Work will be zero as force is perpendicular to displacement.

16. Here, $\vec{r}_1 = (3\hat{i} + 2\hat{j} - 6\hat{k})$ m

$\vec{r}_2 = (14\hat{i} + 13\hat{j} - 9\hat{k})$ m

$\vec{F} = (4\hat{i} + \hat{j} + 3\hat{k})$ N

The displacement from a position $\vec{r}_1$ to position $\vec{r}_2$ is

$\vec{r} = \vec{r}_2 - \vec{r}_1$

$= (14\hat{i} + 13\hat{j} - 9\hat{k}) - (3\hat{i} + 2\hat{j} - 6\hat{k})$

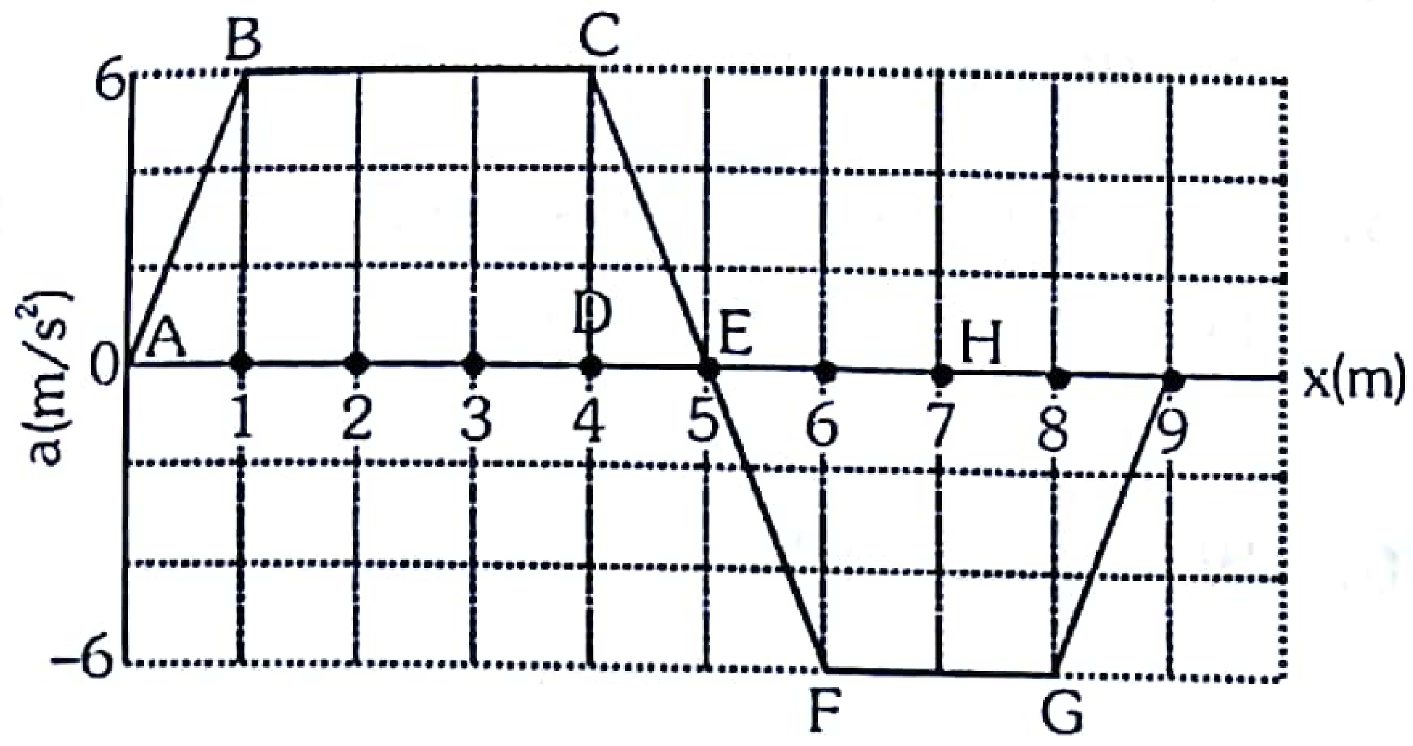
$= 11\hat{i} + 11\hat{j} - 3\hat{k}$

Work done, $W = \vec{F}.\vec{r}$

$= (4\hat{i} + \hat{j} + 3\hat{k}).(11\hat{i} + 11\hat{j} - 3\hat{k})$

$= 44 + 11 - 9 = 46$ J

17.



Here we have acceleration and displacement.

Work done by the force on the body when it reaches at x = 4 m is

W_4 = Mass of the body × Area under ABCD

$$= 2\left[\left(\frac{1}{2}\times1\times6\right)+(3\times6)\right] = 42\text{J}$$

Work done by the force on the body when it reaches at x = 7 m is

$W_7 = W_4$ + (Mass of the body) (Area under CDE – Area under EFGH)

$$= 42\text{ J} + 2\left[\left(\frac{1}{2}\times1\times6\right)-\left(\frac{1}{2}\times1\times6\right)-(1\times6)\right]\text{J}$$

= 42 J + 2 (3 – 3 – 6) J = 42 J – 12 J = 30 J

18. $x = 3t - 4t^2 + t^3$

$$v = \frac{dx}{dt} = 3 - 8t + 3t^2$$

$$WD = \frac{1}{2}mv_4^2 - \frac{1}{2}mv_0^2 = \Delta KE$$

$$WD = \frac{1}{2}\left(\frac{30}{1000}\right)\left[(19)^2 - (3)^2\right] = 5.285\text{ J}$$

19. $WD = \frac{1}{2}mv^2 - \frac{1}{2}mu^2 = \Delta KE$

20. $K \propto p^2$

So if p become 'n' times & K becomes n^2 times

21. $K \propto p^2$

$$\Rightarrow \frac{\Delta K}{K} = \frac{2\Delta p}{p} \qquad \text{So } \frac{\Delta p}{p} = \frac{1}{2};\ \frac{\Delta K}{K} = 1.5\%$$

22. $p \propto \sqrt{K}$

$$\Rightarrow \frac{p_1}{p_2} = \sqrt{\frac{K_1}{K_2}} \Rightarrow p_2 = \sqrt{2}p_1 \text{ as } K_2 = 2K_1$$

$\Rightarrow p_2 = 1.41\ p_1$

So momentum will increase by 41.4%

23. p = constant so $K \propto \frac{1}{m}$

$\Rightarrow K_1 : K_2 = m_2 : m_1 = 5 : 1$

24. $E_k = \frac{p^2}{2m}$

$$\sqrt{E_K} \times \frac{1}{p} = \text{constant}$$

∴ graph is rectangular hyperbola

25. $E_K = \frac{1}{2}mv^2$ ∴ Graph is parabola

26. Work = Change in kinetic energy

$$= E_f - E_i = \frac{1}{2}m(v_f^2 - v_i^2)$$

$$W = \frac{1}{2}(2)(0^2 - 20^2) \Rightarrow W = -400\text{ J}$$

27. $KE = \frac{p^2}{2m}$ ∴ $\frac{1}{4} = \frac{p^2}{p'^2}$

$4KE = \frac{p'^2}{2m}$ ∴ $p' = 2p$

28. Energy dissipated = kinetic energy – potential enrgy

$$\Rightarrow E = \frac{1}{2}mv^2 - mgh$$

$$\Rightarrow E = \frac{1}{2}\times0.5\times(14)^2 - (0.5)(9.8)(8.0)$$

$\Rightarrow E = (49 - 39.2)\text{ J} \Rightarrow E = 9.8\text{ J}.$

29. $KE = \frac{p^2}{2m}$

$2KE = \frac{p'^2}{2m}$ ∴ $\frac{1}{2} = \frac{p^2}{p'^2}$ $p' = \sqrt{2}p$

30. $\frac{KE_1}{KE_2} = \frac{m_1gh}{m_2gh} = \frac{2}{4} = \frac{1}{2}$

31. $\frac{p_1^2}{m_1} = \frac{p_2^2}{m_2}$; $\frac{p_1}{p_2} = \sqrt{\frac{m_1}{m_2}} = \frac{1}{2}$

32. $KE = \frac{p^2}{2m}$

$4KE = \frac{p'^2}{2m}$

$\frac{1}{4} = \left(\frac{p}{p'}\right)^2$

$p' = 2p$

$\therefore$ momentum ↑ by 100%

33. $\frac{2\Delta p}{p} = \frac{\Delta K}{K} \Rightarrow \frac{\Delta p}{p} = 2\%$

34. WD is independent of path for conservative forces.

35. WD is independent of path for conservative forces.

36. WD is independent of path for conservative forces.

37. $\vec{F} = -\frac{dU}{dx}\hat{x}$

38. Viscous force is a non conservative

39. PE is minimum at stable equilibrium

40. $\frac{dU}{dx} = 0$

$16x - 4 = 0$

$x = \frac{1}{4} = 0.25m$

41. For conservative force in a closed loop W = 0

$WD_{PQ} + WD_{QR} + WD_{RP} = 0$

$5 + 2 + WD_{RP} = 0$

$\therefore \quad WD_{PR} = 7J$

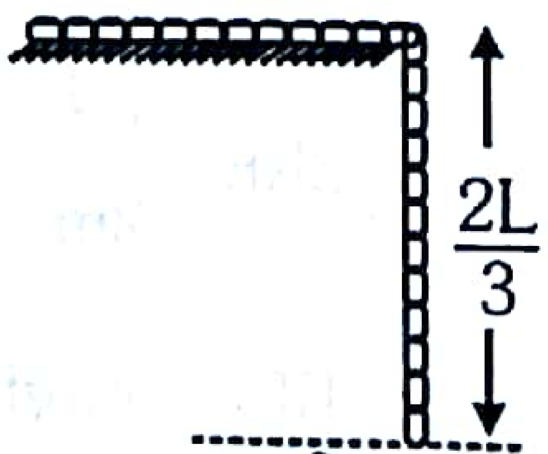
42. As we are pulling the bucket with constant velocity and leakage is at constant rate. We can take average mass

$W = m_{avg}gh = \left(\frac{15+9}{2}\right) \times 10 \times 15 = 1800J$

43. Work done = mgh

m = mass of hanging part

$= \frac{M}{L} \cdot \left(\frac{2L}{3}\right) = \frac{2M}{3}$



h = motion of COM

$= \left(\frac{2L}{3}\right) \cdot \frac{1}{2} = \frac{L}{3}$

$WD = mgh = \frac{2M}{3} g \cdot \frac{L}{3} = \frac{2MgL}{9}$

44. Here gravitation force of earth is centripetal force

$\theta = 90°$, WD = 0

45. Since force is constant so work done is path independent. Hence $W_1 = W_2$

46. $W = \frac{1}{2} k(x_f^2 - x_i^2)$

$= \frac{1}{2}[800\,(0.15)^2 - (0.05)^2] \quad \Rightarrow W = 8J$

47. T = kx for spring

$\text{Energy} = \frac{1}{2}kx^2 = \frac{1}{2}k\frac{T^2}{k^2} = \frac{T^2}{2k}$

48. $2 \times 10 \times 20 = 400\ J$

49. $\frac{1}{2} \times k \times (2)^2 = U$

$\frac{1}{2} \times k \times (10)^2 = U'$

$\therefore \quad \frac{4}{100} = \frac{U}{U'}$

$U' = 25U$

50. $U = \frac{A}{r^{12}} - \frac{B}{r^6}$

$\frac{dU}{dr} = 0$ at Equlibrium

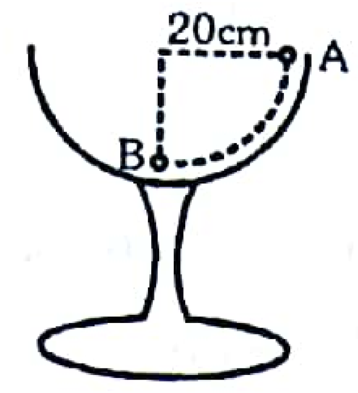
$\therefore \frac{-12A}{r^{13}} - \frac{(-6)B}{r^7} = 0; \quad \frac{6}{r^7}\left[\frac{-2A}{r^6} + B\right] = 0$

$r = \left(\frac{2A}{B}\right)^{1/6}$

$\therefore$ In Eq U is given by

$U = \frac{A}{\left(\frac{2A}{B}\right)^2} - \frac{B}{\frac{2A}{B}} = \frac{B^2}{4A} - \frac{B^2}{2A} = -\frac{B^2}{4A}$

51.



Work done by gravitational force

= force × component of displacement along force

$= mg(R) = \left(\frac{2}{1000}\right) \times (9.8) \times \left(\frac{20}{100}\right)$

$= 392 \times 10^{-5}\ J = 3.92\ mJ$

52. Work done in stretching a spring

$W = \frac{1}{2}kx^2$

where k is the spring constant and x is the extension in the spring

$\therefore W_1 = \frac{1}{2}kx_1^2$ ....(i)

and $W_2 = \frac{1}{2}kx_2^2$ ....(ii)

Divide (ii) by (i), we get

$\frac{W_2}{W_1} = \left(\frac{x_2}{x_1}\right)^2 = \left(\frac{20cm}{10cm}\right)^2 = 4$

$W_2 = 4W_1$

Extra work done $= W_2 - W_1 = 4W_1 - W_1$

$= 3\ W_1 = 3 \times 4\ J = 12\ J$

($\because W_1 = 4J$ (Given))

53. Given : $U = \frac{20xy}{z}$

For a conservative field

$\vec{F} = -\vec{\nabla}U$

Where, $\vec{\nabla} = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial k}$

$\therefore \vec{F} = -\left[\hat{i}\frac{\partial U}{\partial x} + \hat{j}\frac{\partial U}{\partial y} + \hat{k}\frac{\partial U}{\partial z}\right]$

$= -\left[\hat{i}\frac{\partial}{\partial x}\left(\frac{20xy}{z}\right) + \hat{j}\frac{\partial}{\partial y}\left(\frac{20xy}{z}\right) + \hat{k}\frac{\partial}{\partial k}\left(\frac{20xy}{z}\right)\right]$

$= -\left[\hat{i}\left(\frac{20y}{z}\right) + \left(\frac{20x}{z}\right)\hat{j} + \left(-\frac{20xy}{z^2}\right)\right]$

$= -\left(\frac{20y}{z}\right)\hat{i} - \left(\frac{20x}{z}\right)\hat{j} + \frac{20xy}{z^2}\hat{k}$

54. $U = \frac{1}{2}mv^2$

$\therefore \quad m = \frac{2U}{v^2}$

55. $W = mgh$

$= 1 \times 9.8 \times 10 = 98J$

56. At height $= \frac{4h}{5}$

$PE = m \times g\frac{4h}{5}$

$KE = mg \times \frac{h}{5}$

$\therefore \quad KE : PE = 1 : 4$

57. By COME $= \frac{1}{2}mv^2 = \frac{1}{2}kx^2$

$= \frac{1}{2} \times 16 \times 4 \times 4 = \frac{1}{2} \times 100 \times x^2$

$x = \frac{16}{10} = 1.6m$

58. $\frac{1}{2}\times 0.5\times(1.5)^2=\frac{1}{2}\times 50\times x^2$

$\frac{0.5\times(1.5)^2}{50}=x^2$ $\qquad$ x = 0.15m

59. In projectile motion, there is no change in kinetic energy of projectile while landing to the ground and projected from ground. Because speed remains same during projection and striking.

60. Power = $\vec{F}.\vec{v}$

= 4500 × 2 = 9 kW

61. $F=\frac{\text{Power}}{v}=\frac{100\times 750}{72\times\frac{5}{18}}=3.75\times 10^3\text{N}$

62. $P=\frac{mgh}{t}=\frac{300\times 10\times 2}{3}=2000\text{ W}$

for g = 9.8 m/s² P is 1960 W

63. $P_1=\frac{WD_1}{T_1}$

$P_2=\frac{WD_2}{T_2}$ $\qquad$ $\therefore \frac{P_1}{P_2}=\frac{5}{3}\times\frac{9}{11}=\frac{15}{11}$

64. $7\times 10^3=F\times 40\times\frac{5}{18}$

$\therefore$ F = 630 N

65. For the block moving in upward direction

T − 10g = 10a ⇒ T = 10(g + g/2) = 150 N

$s=\frac{1}{2}\left(\frac{g}{2}\right)(2)^2=10\text{ m}$

$P=\frac{W}{t}=\frac{T.s}{t}=\frac{1500}{2}=750\text{W}$

66. $P=mav\Rightarrow P=m\left(v\frac{dv}{dx}\right).v$

$\Rightarrow mv^2dv=Pdx \Rightarrow \frac{mv^3}{3}=Px\Rightarrow v\propto x^{1/3}$

67. Amount of water flowing per unit time $\frac{dm}{dt}=Av\rho$

v = velocity of flow, A is area of cross–section
ρ = density of liquid
To get n times water in the same time,

$\left(\frac{dm}{dt}\right)'=n\frac{dm}{dt}\Rightarrow A\,v'\rho=nAv\rho \Rightarrow v'=nv$

$F=\frac{vdm}{dt}\Rightarrow F'=v'\frac{dm'}{dt}=n^2v\frac{dm}{dt}=n^2F$

To gets n times water, force must be increased n times.

68. Force against which work done is

$F=mg\sin\theta=4\times 9.8\times\frac{1}{40}=0.98\text{ N}$

speed v = 40 m/s
for 50% efficiency required power = 2 (F·v)

69. Mass of water $=2238\times 10^{-3}\times 10^3$
$= 2238$ kg

$\therefore$ Energy = 2238 × 10 × 10 = mgh

$\therefore \frac{2238\times 30\times 10}{T}=1\times 750$ (T is time)

$\therefore T=\frac{2238\times 10\times 10}{750}$ second = 5 min.

70. Power = 100 × 10 × 100 = 100 kW

71. $a=\frac{F}{m};\ s=\frac{1}{2}at^2\ \&\ v=at$

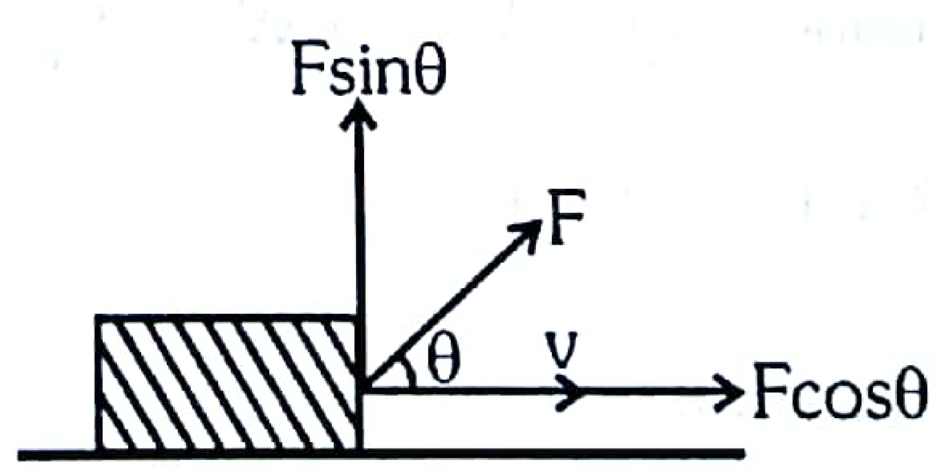
Average power (P) $=\frac{W}{t}=\frac{F.s}{t}=\frac{F\left(\frac{1}{2}at^2\right)}{t}=\frac{1}{2}F.v$

72. Power = $\frac{\text{work done as change in PE}}{\text{time}}$

$\therefore P=\frac{mgh}{t}=\frac{80\times 10\times 6}{10}=480\text{ W}$

$\therefore P=\frac{480}{746}\text{hp}=0.63\text{ hp}$

73.



Power = $\vec{F}.\vec{v}$ = Fv cos θ

12. $\vec{F}.\vec{v} = P_0$

$$m\frac{dv}{dt} \times v = P_0$$

$$\frac{v^2}{2} = \frac{P_0 t}{m}$$

$$v \propto \sqrt{\frac{t}{m}} \propto t^{1/2}$$

13. $W = \vec{F}.\vec{s} = (3\hat{i}+\hat{j}).[(4-2)\hat{i}+(3-0)\hat{j}+(-1-1)\hat{k}]$

$= (3\hat{i}+\hat{j}).(2\hat{i}+3\hat{j}-2\hat{k})$

$= 3(2) + 1(3) + 0(-2) = 9\ J$

14. $W = -\int F dx$

$$W = -\int_{20}^{30} 0.1x\, dx$$

$$W = -0.1\left[\frac{x^2}{2}\right]_{20}^{30}$$

$$W = -0.1\left[\frac{900-400}{2}\right] = -25$$

From work energy theorem $W = K_f - K_i$

$$\Rightarrow -25 = K_f - \frac{1}{2}\ 10(10)^2$$

$$\Rightarrow K_f = 475$$

15. $P = Fv = mav$

$$\Rightarrow k = mv\frac{dv}{dt}$$

By integrating the equation

$$\Rightarrow \int v\,dv = \int \frac{k}{m}dt$$

$$\Rightarrow \frac{v^2}{2} = \frac{k}{m}t \Rightarrow v = \sqrt{\frac{2k}{m}t}$$

$$a = \frac{dv}{dt} = \sqrt{\frac{2k}{m}}\left(\frac{1}{2}t^{-1/2}\right)$$

$$F = ma = m\left(\frac{1}{2}\right)\sqrt{\frac{2k}{mt}} \Rightarrow F = \sqrt{\frac{mk}{2t}}$$

16. Pressure = 150 mm Hg

Pumping rate $= \frac{dV}{dt} = \frac{5\times10^{-3}}{60}\ m^3/s$

Power of heart $= P.\frac{dV}{dt} = \rho gh \times \frac{dV}{dt}$

$$= (13.6\times10^3\ kg/m^3)(10)\times(0.15)\times\frac{5\times10^{-3}}{60}$$

$$= \frac{13.6\times5\times0.15}{6} = 1.70\ watt$$

17. From $KE = \frac{p^2}{2m} = mgh$

$$p = \sqrt{2m^2gh} = \sqrt{2\times(50)^2\times10\times0.8}$$

$$p = 200\ kg\ m/s$$

18. $\vec{F} = 2t\hat{i}+3t^2\hat{j} \Rightarrow m\frac{d\vec{v}}{dt} = 2t\hat{i}+3t^2\hat{j}$ {m = 1 kg}

$$\Rightarrow \int_0^{\vec{v}} d\vec{v} = \int_0^t (2t\hat{i}+3t^2\hat{j})dt \Rightarrow \vec{v} = t^2\hat{i}+t^3\hat{j}$$

Power $= \vec{F}.\vec{v} = (2t^3 + 3t^5)W$

19. $\vec{s} = \vec{r}_f - \vec{r}_i = 2\hat{i}-\hat{j}+3\hat{k}$

$$W = \vec{F}.\vec{s} = (4\hat{i}+3\hat{j}).\left[2i-\hat{j}+3\hat{k}\right] = 8-3 = 5J$$

EXERCISE-III (Analytical Questions) — Check Your Understanding

1. $\vec{d} = (3-2)\hat{i} + (3-1)\hat{j} + (4-3)\hat{k} = \hat{i} + 2\hat{j} + \hat{k}$ and

$\vec{F} = |F|\hat{F}$

So, $\vec{F} = 20\left[\frac{1}{\sqrt{6^2+8^2}}(6\hat{i}+8\hat{j})\right] = 12\hat{i} + 16\hat{j}$

$W = \vec{F}.\vec{d} = 44J$

2. $a = \frac{v}{t_1}$ & $F = ma = \frac{mv}{t_1}$

$s = \frac{1}{2}at^2 \Rightarrow s = \frac{1}{2}\frac{v}{t_1}t^2$

$W = F.s = \frac{1}{2}m\frac{v^2}{t_1^2}t^2$

3. $K_f = \frac{1}{4}K_i \Rightarrow v_f = \frac{v_0}{2}$

$a = \mu g$ [as $f = \mu mg$]

So $\frac{v_0}{2} = v_0 - \mu_k g\, t_0 \Rightarrow \mu = \frac{v_0}{2gt_0}$

4. $P = F.v = ma.v$

$a = \frac{v_1}{t_1}$ & $v = 0 + \frac{v_1}{t_1}t$

So $P = m\left(\frac{v_1}{t_1}\right)\left(\frac{v_1}{t_1}t\right) \Rightarrow P = \frac{mv_1^2}{t_1^2}t$

5. Till x = 2m, area under the curve F – d is zero so W.D. is zero therefore KE remains same at x = 2m, v = 4 m/s

Force = –4 N, mass of body = 2 kg

$\therefore$ acceleration (a) = $\frac{-4N}{2kg} = -2\ m/s^2$.

This reduces velocity.

Now $v^2 = u^2 + 2as$.

$\therefore v^2 = (4)^2 + 2\times(-2) = (16-4) = 12$

at x = 3 m and onwards.

$\therefore$ Kinetic energy = $\frac{1}{2}\times 2\times 12 = 12$ J

6. Applying law of conservation of energy

$\frac{1}{2}mv^2 = \frac{75}{100}\times 12$

or $v = \sqrt{\frac{75\times12\times2}{100\times m}} = \sqrt{\frac{3}{4}\times\frac{12\times2}{1}} = \sqrt{18}\ m/s.$

7. Force = $\frac{-A}{R^2}$

$\therefore$ Potential energy = $-\int_{\infty}^{R} FdR = \frac{-A}{R}$

K.E. = $\frac{1}{2}\frac{A}{R}$ by $F_{centripetal} = \frac{mv^2}{R}$

T.E. = $\frac{-A}{2R}$

8. $\vec{F} = 3x^2\hat{i} + 4\hat{j}$

$\vec{r} = x\hat{i} + y\hat{j} \Rightarrow d\vec{r} = dx\hat{i} + dy\hat{j}$

Work done, $W = \int \vec{F}.d\vec{r}$

$= \int_{(2,3)}^{(3,0)} (3x^2\hat{i} + 4\hat{j}).d\vec{r}$

$= \int_{(2,3)}^{(3,0)} 3x^2dx + 4dy$

$= [x^3 + 4y]_{(2,3)}^{(3,0)} = 3^3 + 4\times0 - (2^3 + 4\times3)$

$= 27 + 0 - (8 + 12) = 27 - 20 = +7$ J

According to work energy theorem,

Change in the kinetic energy = Work done

$\Delta KE = +7$ J